

Contents

Summary	9
Streszczenie	10
Acknowledgement	11
Notations and symbols	13
1 Introduction	15
2 A linear retarded type time delay system	18
2.1 Preliminaries	18
2.2 The Lyapunov functional for a linear system with one delay	22
2.2.1 Mathematical model of a linear time delay system with one delay	22
2.2.2 Determination of the Lyapunov functional	24
2.2.3 The examples	28
2.2.3.1 Inertial system with delay and a P controller	28
2.2.3.2 Inertial system with delay and an I controller	32
2.3 The Lyapunov functional for a linear system with two delays	42
2.3.1 Mathematical model of a linear time delay system with two delays	42
2.3.2 Determination of the Lyapunov functional	43
2.3.3 Solution of the set of differential equations (2.170) for commensurate delays	47
2.3.4 The example	50
2.4 A linear system with both lumped and distributed retarded type time delay	52
2.4.1 Mathematical model of a linear system with both lumped and distributed retarded type time delay	52
2.4.2 Determination of the Lyapunov functional	53
2.4.3 The examples	58
2.4.3.1 The example 1	58
2.4.3.2 The example 2	64
2.5 A linear system with a retarded type time-varying delay	71
2.5.1 Mathematical model of a linear system with a retarded type time-varying delay	71
2.5.2 Determination of the Lyapunov functional	72

2.5.3	The examples	76
2.5.3.1	Inertial system with delay and a P controller	76
2.5.3.2	The example. Two dimensional system	81
3	A linear neutral system	90
3.1	Preliminaries	90
3.2	A linear neutral system with lumped delay	93
3.2.1	Mathematical model of a linear neutral system with lumped delay	93
3.2.2	Determination of the Lyapunov functional for a neutral system with one delay	95
3.2.3	The example. Inertial system with delay and a PD controller	98
3.3	The Lyapunov functional for a neutral system with both lumped and distributed time delay	101
3.3.1	Mathematical model of a linear neutral system with both lumped and distributed time delay	101
3.3.2	Determination of the Lyapunov functional coefficients	103
3.3.3	The example	109
3.4	A linear neutral system with a time-varying delay	111
3.4.1	Mathematical model of a linear neutral system with a time-varying delay	111
3.4.2	Determination of the Lyapunov functional	112
3.4.3	The example. Inertial system with delay and a PD controller	118
4	The Lyapunov matrix for a retarded type time delay system	124
4.1	Mathematical model of a retarded type time delay system	124
4.2	The Lyapunov–Krasovskii functional for a retarded type time delay system	125
4.3	The Lyapunov matrix for a system with one delay	127
4.4	Formulation of the parametric optimization problem for a system with one delay	129
4.5	The examples	130
4.5.1	Inertial system with delay and a P-controller	130
4.5.2	Inertial system with delay and a PI-controller	134
4.6	The Lyapunov matrix for a system with two commensurate delays	140
4.7	Formulation of the parametric optimization problem	144
4.8	The example. Parametric optimization problem for a scalar system with two delays	145
5	The Lyapunov matrix for a neutral system	149
5.1	The Lyapunov matrix for a neutral system with one delay	149
5.1.1	Mathematical model of a neutral system with one delay	149
5.1.2	The Lyapunov–Krasovskii functional for a neutral system with one delay	151
5.1.3	The Lyapunov matrix for a neutral system with one delay	153
5.1.4	Formulation of the parametric optimization problem for a neutral system with one delay	155
5.1.5	The examples	156
5.1.5.1	A linear neutral system with a P-controller	156
5.1.5.2	Inertial system with delay and a PD-controller	160
5.2	Neutral system with two delays	165
5.2.1	Mathematical model of neutral system with two delays	165

5.2.2 The Lyapunov–Krasovskii functional for a neutral system with two delays 167

5.2.3 Formulation of the parametric optimization problem for a neutral system
with two delays 169

5.2.4 The Lyapunov matrix for a neutral system with two delays 169

5.2.5 The Lyapunov matrix for a neutral system with two commensurate delays 172

5.2.6 The example 176

6 Conclusion 181

Bibliography 184